# TIME-DOMAIN ASTRONOMY

## Lectures 7: Time of Arrival Analysis

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# Time of Arrival

- So far, we have assumed that the our data are constituted by a set of $N$ data points $(t_j, y_j)$, $j = 1,...,N$, possibly with known errors for $y$.

- We can anyway think to different datasets. For instance, at X-ray and shorter wavelengths, individual photons are detected and background contamination is often negligible.
  - In such cases, the data set consists of the arrival times of individual photons $t_j$, $j=1,...,N$, or, in principle and more generally, the time of occurrence of a given phenomenon.

- Given such a data set, how do we search for a periodic signal, and more generally, how do we test for any type of variability?

# Rayleigh Test

- The best known classical test for variability in arrival time data is the Rayleigh test.

- Given a trial period, the phase $\varphi_j$ corresponding to each datum is evaluated using:

$$\phi = \frac{t}{P} - \mathrm{int}\left(\frac{t}{P}\right),$$

- And the following statistics is computed:

$$R^2 = \left(\sum_{j=1}^{N} \cos(2\pi\phi_j)\right)^2 + \left(\sum_{j=1}^{N} \sin(2\pi\phi_j)\right)^2.$$

# Rayleigh Test

- A simple way to read this expression is referring to a random walk.

- Each angle $\varphi_j$ defines a unit vector, and $R$ is the length of the resulting vector.

$$R^2 = \left(\sum_{j=1}^{N} \cos(2\pi\phi_j)\right)^2 + \left(\sum_{j=1}^{N} \sin(2\pi\phi_j)\right)^2.$$

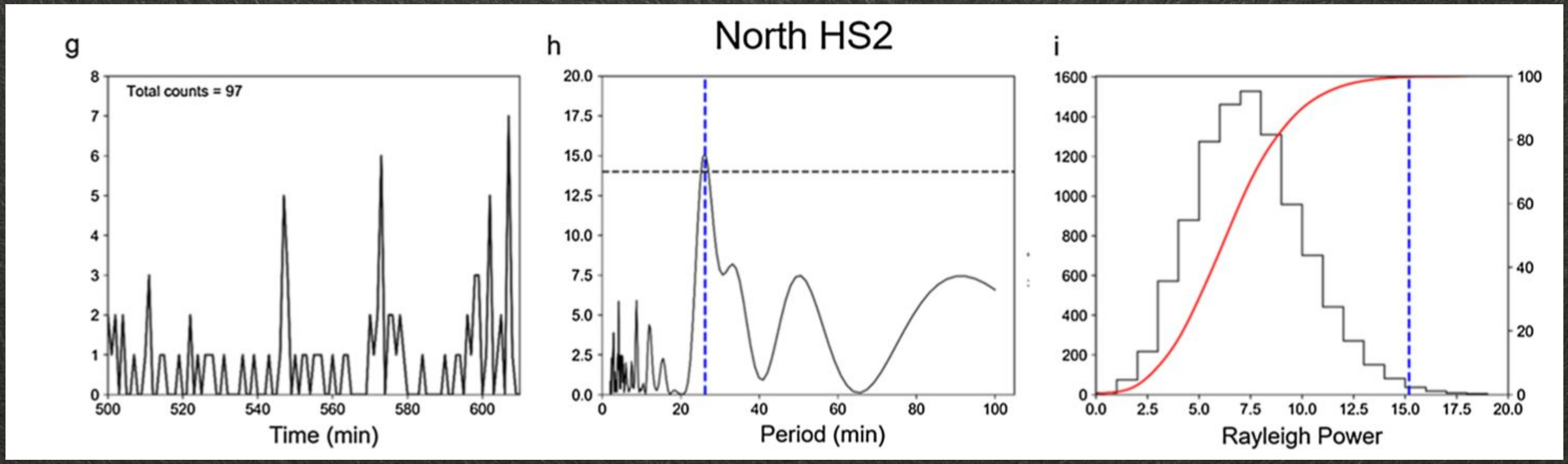
- For random data $R^2$ is small, and for periodic data $R^2$ is large when the correct period is chosen.

- $R^2$ is evaluated for a grid of $P$, and the best period is chosen as the value that maximizes $R^2$.

# Rayleigh Test

- For $N > 10$, $2R^2/N$ is distributed as $\chi^2$ with two degrees of freedom (this easily follows from the random walk interpretation).

- Then, one can assess the significance of the best-fit period as the probability that “a value that large” would happen by chance when the signal is stationary.




From Weigt et al. (2017) - Chandra Observations of Jupiter's X-ray Auroral Emission During Juno Apojove 2017

# Rayleigh Test: Multiple Harmonics

- It is possible to generalize the Rayleigh test including multiple harmonics in the time of arrival decomposition:

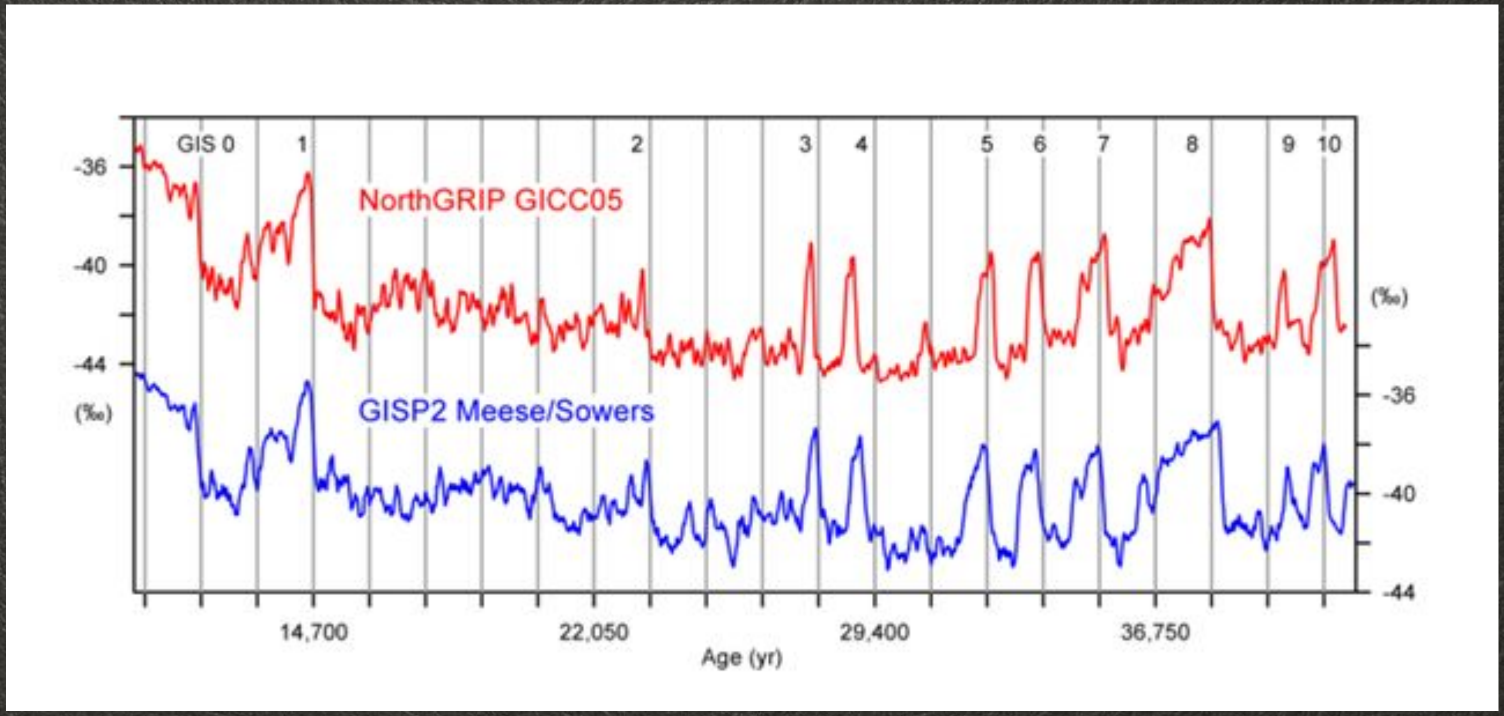
$$Z_n^2 = \frac{2}{N} \cdot \sum_{k=1}^{n} \left[ \left( \sum_{j=1}^{N} cos(k\phi_j) \right)^2 + \left( \sum_{j=1}^{N} sin(k\phi_j) \right)^2 \right]$$

- In the literature the Rayleigh test is often indicated with $Z^2{}_n$, where n is the number of harmonics.

# Exercise

- Useful notebook:

1. DO-climate- events



# Poisson Distribution: a brief recap

- A Poisson Process is a model for a series of discrete event where the average time between events is known, but the exact timing of events is random. The arrival of an event is independent of the event before (waiting time between events is memoryless).
- A Poisson Process meets the following criteria (in reality many phenomena modeled as Poisson processes don't meet these exactly):
  1. Events are **independent** of each other. The occurrence of one event does not affect the probability another event will occur.
  2. The average rate (events per time period) is constant.
  3. Two events cannot occur at the same time.

The last point means an event either happens or not..

# Poisson Distribution: a brief recap

- A Common examples of Poisson processes are customers calling a help center, visitors to a website, radioactive decay in atoms, photons arriving at a space telescope, and movements in a stock price.

- Poisson processes are generally associated with time, but they do not have to be. In the stock case, we might know the average movements per day (events per time), but we could also have a Poisson process for the number of trees in an acre (events per area).

- One instance frequently given for a Poisson Process is bus arrivals (or trains or Ubers). However, this is not a true Poisson process because the arrivals are not independent of one another. Even for bus systems that do not run on time, whether or not one bus is late affects the arrival time of the next bus.

# Poisson Distribution

- The Poisson Distribution probability mass function gives the probability of observing **k** events in a time period given the length of the period and the average events per time:

$$P(\mathrm{k\ events\ in\ time\ period}) = \mathrm{e}^{-\frac{\mathrm{events}}{\mathrm{time}} \times \mathrm{time\ period}} \times \frac{(\frac{\mathrm{events}}{\mathrm{time}} \times \mathrm{time\ period})^{\mathrm{k}}}{\mathrm{k}!}$$

- events/time * time period is usually simplified into a single parameter, **λ**, lambda, the rate parameter:

$$P(\mathrm{k\ events\ in\ interval}) = \mathrm{e}^{-\lambda} \times \frac{\lambda^{\mathrm{k}}}{\mathrm{k}!}$$

- λ can be thought of as the *expected number of events in the interval.*

# Waiting time

- The probability of waiting a given amount of time between successive events decreases exponentially as the time increases. The following equation shows the probability of waiting more than a specified time:

$$P(T > t) = e^{-\frac{\text{events}}{\text{time}} \times t}$$

- Conversely, the probability of waiting less than or equal to a time:

$$P(T \leq t) = 1 - e^{-\frac{\text{events}}{\text{time}} \times t}$$

# Gregory & Loredo (1992) Algorithm

- Let's divide the time interval $T = t_N - t_1$ into many arbitrarily small steps, $\Delta t$, so that each interval contains either 1 or 0 detections.

- Given the event rate $r(t)$, then the expectation value for the number of events during $\Delta t$ is:

$$\mu(t) = r(t)\,\Delta t.$$

- Poisson statistics tells us that the probability of detecting no events during $\Delta t$ is:

$$p(0) = e^{-r(t)\Delta t},$$

# Gregory & Loredo (1992) Algorithm

- While the probability of detecting a single event is:

$$p(1) = r(t)\,\Delta t\, e^{-r(t)\Delta t}.$$

- Given these ingredients we can compute the data likelihood:

$$p(D|r, I) = (\Delta t)^N e^{-\int_{(T)} r(t)\,dt} \prod_{j=1}^{N} r(t_j).$$

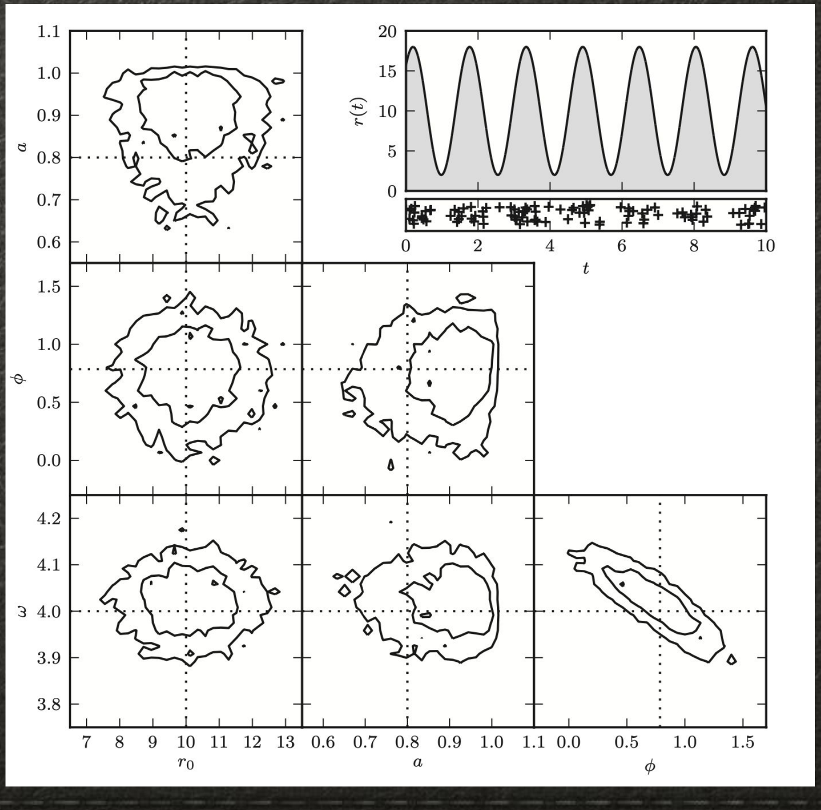
- For simplicity, we assume that the data were collected in a single stretch of time with no gaps. But it is possible to generalize the algorithm.

# Gregory & Loredo (1992) Algorithm

- With an appropriate $r(t)$, and priors for the model parameters, analysis of arrival time data is no different than any other model selection and parameter estimation problem.

- For example, we see here the posterior pdf for a model based on periodic $r(t)$ and arrival times for 104 photons.

$$r(t) = r_0[1 + a\sin(wt + \phi)]$$



# Gregory & Loredo (1992) Algorithm

- Instead of fitting a parametrized model, such as a Fourier series, Gregory and Loredo used a nonparametric description of the rate function $r(t)$.

- They described the shape of the phased light curve using a piecewise constant function, $f_j$, with M steps of the same width, and $\sum_j f_j = 1$.

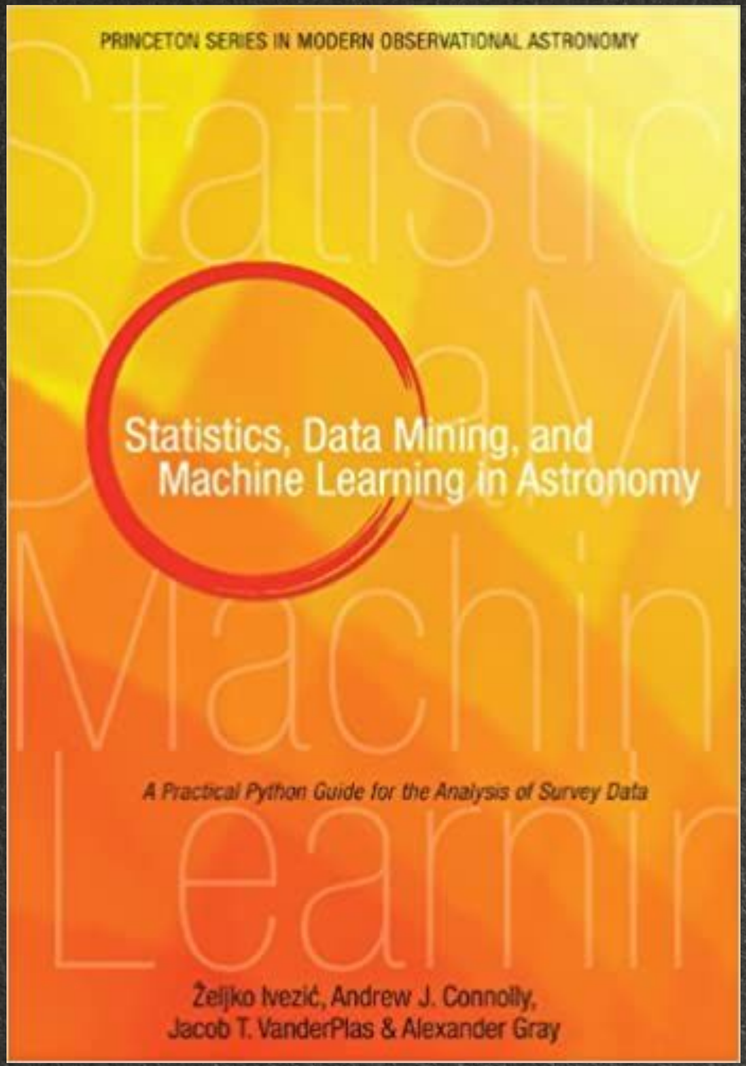
- The rate is therefore described as: $$r(t_j) \equiv r_j = M\,A\,f_j,$$

- where $A$ is the average rate, and bin $j$ corresponding to $t_j$, determined from the phase corresponding to $t_j$ and the trial period.

# Gregory & Loredo (1992) Algorithm

- The model includes the frequency $\omega$ (or period), a phase offset, the average rate $A$, and $M-1$ parameters $f_j$.
- Marginalizing the resulting pdf one can produce i) an analog of the periodogram, ii) expressions for computing the model odds ratio for signal detection, and iii) for estimating the light curve shape.
- In the case when little is known about the signal shape, this method is superior to the more popular Fourier series expansion.

# REFERENCES AND DEEPENING

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JGR Planets

RESEARCH ARTICLE
10.1029/2019JE006262

**Special Section:**
Jupiter Midway Through the Juno Mission

**Key Points:**
- Analysis is performed on a Chandra campaign observing Jupiter's X-ray emissions during the apojove portion of Juno's orbit on 18 June 2017
- We use Juno magnetopause crossings to infer a compressed magnetosphere during the X-ray campaign
- Quasiperiodic pulsing of the northern X-ray hot spot are mapped to a location close to the dayside magnetopause

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## Chandra Observations of Jupiter's X-ray Auroral Emission During Juno Apojove 2017

**D. M. Weigt[1], C. M. Jackman[1,2], W. R. Dunn[3,4,5], G. R. Gladstone[6,7], M. F. Vogt[8], A. D. Wibisono[3,4], G. Branduardi-Raymont[3,4], D. Altamirano[1], F. Allegrini[6,7], R. W. Ebert[6,7], P. W. Valek[6], M. F. Thomsen[9], G. Clark[1,10], and R. P. Kraft[5]**

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**Abstract** Jupiter's auroral X-rays have been observed for 40 years with an unknown driver producing quasiperiodic emission, concentrated into auroral hot spots. In this study we analyze an ~ 10-hr Chandra observation from 18:56 on 18 June 2017. We use a new Python pipeline to analyze the auroral morphology, perform timing analysis by incorporating Rayleigh testing, and use in situ Juno observations to infer the magnetosphere that was compressed during the Chandra interval. During this time Juno was near its apojove position of ~112 $R_J$, on the dawn flank of the magnetosphere near the nominal magnetopause position. We present new dynamical polar plots showing an extended X-ray hot spot in the northern auroral region traversing across the Jovian disk. From this morphology, we propose setting a numerical threshold of >7 photons per 5° System III longitude × 5° latitude to define a photon concentration of the northern hot spot region. Our timing analysis finds two significant quasiperiodic oscillations (QPOs) of ~37 and ~26 min within the extended northern hot spot. No statistically significant QPOs were found in the southern X-ray auroral emission. The Rayleigh test is combined with Monte Carlo simulation to find the statistical significance of any QPOs found. We use a flux equivalence mapping model to trace the possible origin of the QPOs, and thus the driver, to the dayside magnetopause boundary.

Zeljko Izevic et al.

Dale Weigt